Four mass coupled oscillator guitar model

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Coupled oscillator models have been used for the low frequency response (50 to 250 Hz) of a guitar. These 2 and 3 mass models correctly predict measured resonance frequency relationships under various laboratory boundary conditions, but did not always represent the true state of a guitar in the players' hands. The model presented has improved these models in three ways, (1) a fourth oscillator includes the guitar body, (2) plate stiffnesses and other fundamental parameters were measured directly and effective areas and masses used to calculate the responses, including resonances and phases, directly, and (3) one of the three resultant resonances varies with neck and side mass and can also be modeled as a bar mode of the neck and body. The calculated and measured resonances and phases agree reasonably well. © *2012 Acoustical Society of America*. [DOI: 10.1121/1.3652849]

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I. INTRODUCTION

A previous two-mass model treated the air oscillating in and out of the sound hole (air piston) and the top like a base reflex speaker (Caldersmith, 1978 and Christensen and Vistisen, 1980). A later three-mass model included the back (Christensen, 1982), but again did not measure or calculate fundamental parameters. These lumped parameter models used measured resonances of isolated oscillators (like air alone, or isolated top plate) to calculate effective areas, effective masses, and/or stiffnesses or their ratios and then predicted how these oscillators would couple and interact. Several relationships between resonant and antiresonant frequencies were postulated for a variety of boundary conditions and compared favorably to experiments in these published articles and verified in others (Rossing, Popp, and Polstein, 1985). A four-mass model, described in Sec. II, adds another degree of freedom by including the sides and neck (referred to hereafter as the ribs) as one more oscillator free to interact with the previous three. Measured and calculated resonances for the three-mass and four-mass models are compared. Four-mass model equations are developed in Sec. III followed by a description of how plate fundamental parameters were measured and/or calculated in Sec. IV. Model predictions for the fundamental resonances of isolated oscillators to a harmonic driving force is followed by predicted response curves for the four-mass coupled oscillator system in Sec. V. Next it is shown that the frequency of one of the low frequency resonances is strongly dependent upon rib mass and in Sec. VI whole guitar "bar" modes are offered as an alternative model for explaining this dependence.

II. MODEL COMPARISONS

In the frequency range from about 90 to 240 Hz, there are typically three (0,0) modes, i.e., modes with no nodes except at or near the plate edges. One of these involves motion of the ribs and/or the neck and is sensitive to the mass of these components. A comparison of three-mass and this four-mass model demonstrates the need for this fourth oscillator.

A. The four-mass model

The present model adds a fourth oscillator (guitar sides and neck, or "ribs," x_r), as shown in Fig. 1. It uses measured or calculated top and back plate areas, masses and stiffnesses, cavity volume, and sound hole area and other fundamental parameters and applies Newton's second law to predict both the fundamental resonances of "isolated" system oscillators and the (0,0) mode resonances and phases when the four oscillators interact freely as is the case for a played instrument.

B. Measured resonances for six guitars

In most of the experiments in this paper, the guitar was driven at some selected point *P* (usually at or near the bridge) with a Bruel and Kjaer (B&K) 4810 vibration exciter used in conjunction with an 8001 impedance head, which has an effective mass load of 2.1 g. The output from the force transducer was fed to a GenRad 1569 automatic level regulator and an audio amplifier to provide a constant driving force. The accelerometer output was amplified and integrated by a B&K 2651 charge amplifier. This velocity signal was fed through a tracking filter/amplifier (GenRad 1901) to a chart recorder (GenRad 1521) in order to obtain a graph of driving point mobility (v/F at point *P*) vs frequency. The transfer mobility (v/F) was obtained by affixing a BBN 501 accelerometer ($m \sim 2$ g) at *Q* and driving the guitar at constant force, as described above. A B&K 2971 phase meter was used to determine the phase of the velocity and sound pressure with respect to the driving force.

Six guitars (all classical except the Penades flamenco, and the D28 steel-string) examined show significant changes in the measured resonant frequencies as we move from the clamped ribs to the free ribs boundary condition (Table I). Clamping was accomplished by placing the guitars in a vertical frame that held a truck tire inner tube filled with sand completely around the guitar perimeter with double sticky

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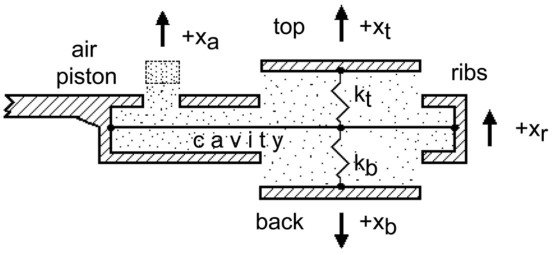

FIG. 1. The top and back plates couple to the ribs (neck and sides) through the plate stiffnesses and the top, back, and air piston through the air in the cavity. Positive directions illustrate "in-phase" motion, i.e., top and back plates are in phase when they both oscillate simultaneously away from the c.m. (and each other) and inward together 1/2 cycle later.

TABLE I. Measured resonances for six guitars for two sets of boundary conditions.

Guitar	Clamped ribs[a] freq. (Hz)	Free ribs[b] freq. (Hz)
D28	104, 167, 207	104, …, 197
Kohno	103, 181, 231	104, 215, 231
Krempel	103, 179, …	105, 208, …
Framus	116, …, 210	118, …, 235
Kingston	100, 154, 219	101, 187, 230
Penade's	94, 178, 229[c]	94, …, 213

[a]Clamped ribs results are similar to three-mass model predictions.
[b]Free ribs results are similar to played guitar and to four-mass model predictions.
[c](0,0) top and (0,1) back.

tape between the tube and the sides and the free state was accomplished by supporting the guitar by rubber bands. For all six, the second resonance is either shifted upward, by about 30 Hz, or disappears entirely as the boundary condition is changed from free to clamped. Previous models assumed fixed ribs, yet a considerable mass load is needed to immobilize the ribs of a guitar. This requirement is not met when the guitar is in the players hands. Thus, a proper model of the guitar should allow for the ribs to move freely, and guitars should be driven while freely supported if one wishes to test the guitar under playing conditions.

C. Calculated and measured resonances

The previous three-mass model assumed that the ribs and the plate edges do not move. This assumption is equivalent to assuming that the rib mass is infinite, and corresponds most closely to experiments with rigidly clamped sides and/or neck. The four-mass model, instead, calculated the resonances using a finite rib mass (calculated from measured thickness, area and known density). These calculated resonances correspond more closely to those measured for a ("free") guitar supported by rubber bands than the three-mass model calculations (Table II).

Because assigning the ribs a finite (rather than an infinite) mass changes the boundary conditions, but does not introduce an additional degree of freedom, the four-mass model does not introduce any new (0,0) mode resonances, beyond the three that the earlier three-mass model described (Christensen, 1982). It does alter or eliminate one the of the three (0,0 mode) frequencies from what is predicted by both the earlier three-mass model and this "reduced" four-mass model.

III. MODEL EQUATIONS

Newton's second law of motion applied to the top plate requires expressions for each of four forces acting upon it. For the top plate they are, in the order presented in the first expression of Eq. (1), the applied harmonic force ($F_o e^{jwt}$), the force due to top plate stiffness (k_t), the force due to pressure changes (Δp) inside the guitar cavity, and the force due to viscous friction $\left(R_t \dot{x}_t\right)$. Similar equations apply to the other three oscillators.

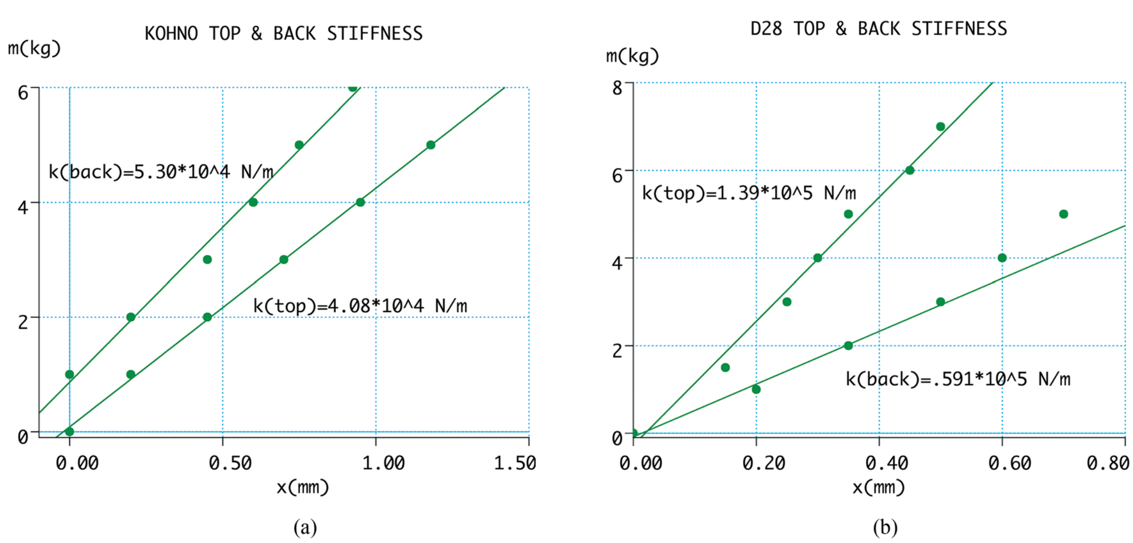

FIG. 2. (Color online) (a) The stiffness in this typical Kohno classical guitar is larger in the back than in the top. (b) The Martin cross braced folk guitar is nearly three times stiffer in the top than the Kohno classical, has a relatively soft back, and shows an increasing stiffness with increasing load. The stiffness (slope) was calculated from the first four points since displacements for the plates of a driven or played guitar are typically smaller than these measurements.

TABLE II. Calculated and measured resonances for two guitars.

Models		Kohno res. freq. (Hz)	Rib mass (kg) or bound. cond.	D28 res. freq. (Hz)	Rib mass (kg) or bound. cond.
4 mass model	Calc.	105, 206, 234	0.433	102,…, 200	0.45
	Meas.	104, 215, 231	Free	104,…, 197	Free
3 mass model	Calc.	103, 173, 216	Infinite[a]	102, 166, 197	Infinite[a]
	Meas.	103, 181, 231	Clamped ribs	104, 167, 207	Clamped ribs
Bar modes	Calc.	84, 233, 579	Free-free		
	Meas.	091, 272, 353	see note[b]	53, 200	see note[b]

[a]Three- mass model predictions are calculated by assuming a large rib mass in the four-mass model. The same result occurs if the 4×4 matrix used in the four-mass model is reduced to a 3×3 matrix which excludes the rib mass. The same result can also be achieved by increasing the rib mass to an arbitrarily large value.

[b]Bar modes were measured with loose styrofoam in the cavity, rose hole closed, sound posts between plates, rubber band supports, and driven at the tail block and are described in Sec. VI.

$$\begin{aligned} m_t\ddot{\tilde{x}}_t &= F_o e^{jwt} - k_t(\tilde{x}_t - \tilde{x}_r) + \Delta p A_t - R_t\dot{\tilde{x}}_t \\ m_b\ddot{\tilde{x}}_b &= -k_b(\tilde{x}_b + \tilde{x}_r) + \Delta p A_b - R_b\dot{\tilde{x}}_b, \\ m_r\ddot{\tilde{x}}_r &= +k_t(\tilde{x}_t - \tilde{x}_r) - k_b(\tilde{x}_b + \tilde{x}_r), \\ m_a\ddot{\tilde{x}}_a &= +\Delta p A_b - R_b\dot{\tilde{x}}_b. \end{aligned} \tag{1}$$

The subscripts used refer to the top (t), back (b), ribs (r), and air piston (a). The complex displacements, velocities and accelerations are represented by $\tilde{x}_i$, $\dot{\tilde{x}}_i$, and $\ddot{\tilde{x}}_i$, respectively, where $i = t, b, r$, or a.

The ideal gas law and the law for adiabatic expansion can be used to express the force due to pressure oscillations inside the guitar in terms of cavity volume changes

$$pV^\gamma = \text{const.}, \quad \text{so} \quad \Delta p = -p\gamma \Delta V/V, \tag{2}$$

where p and V are the pressure and volume inside the cavity, and γ is the ratio of specific heats. Also

$$p = \rho \mathrm{kT/m} \quad \text{and} \quad c^2 = \gamma \mathrm{kT/m}, \tag{3}$$

with $\rho =$ air density, $k =$ Boltzman's constant, $T =$ absolute temperature, $m =$ cavity air mass, and $c =$ velocity of sound in air. Combining Eqs. (2) and (3) gives

$$\Delta p = -g\Delta V, \quad \text{where} \quad g = \rho c^2/V. \tag{4}$$

It is evident from the geometry of the model that

$$\Delta V = A_t(\tilde{x}_t - \tilde{x}_r) + A_b(\tilde{x}_b + \tilde{x}_r) + A_a(\tilde{x}_a - \tilde{x}_r). \tag{5}$$

Four linear equations in the complex displacements results after (1) substituting Eqs. (4) and (5) into Eq. (1), (2) assuming complex harmonic solutions ($\tilde{x}_i = x_i e^{jwt}$), (3) performing the time derivatives, and (4) factoring out the exponential time part of the solution. The four complex displacements can now be calculated using matrices and determinants. Displacement and/or velocity amplitudes and phases of each oscillator, relative to a unit driving force and as a function of driving force frequency, can be calculated and plotted using computer software.

IV. FUNDAMENTAL PARAMETERS

The values of the parameters in Eqs. (1)–(5) unique to each guitar must be determined in order to predict response functions, resonant frequencies, and phases. While some of these, like sound hole area and cavity volume are self evident, plate stiffness measurements, and effective plate mass and effective plate area calculations need further explaining.

A. Sound hole air piston mass and stiffness

The guitar is a Helmholtz resonator if the top, back, and neck are rigidly clamped or weighted. The air piston can then be modeled as a cylinder of volume V_a, radius r and height H:

$$V_a = \pi r^2 H, \quad \text{where} \quad H = 2\alpha r \tag{6}$$

with $\alpha = 0.80$ for a hole in a thin walled resonator (Kinsler *et al.*, 2000). From density, $\rho = m_a/V_a$, where $m_a =$ piston mass, so

$$m_a = 2\pi\alpha\rho r^3. \tag{7}$$

B. Plate stiffness

Forces were applied directly to the top plate at its most compliant position at or near the bridge center and over a 1 cm^2 area. Displacements at this point were measured relative to the plate edge (guitar sides or "ribs") and a graph of displacement vs force made. The slope yields the static stiffness constants (k_t and k_b) for the top and back plates (see Fig. 2).

C. Effective plate area and mass

Guitar top and back plates oscillation amplitudes vary throughout their surfaces. This model replaces these flat plates (curved, of course, while oscillating) with an equivalent flat circular piston that has a smaller (effective) area and (effective) mass than the actual plates. The criteria used to calculate the area and mass of this equivalent oscillator are that (1) the change in air cavity volume is the same for the equivalent piston as for the plates and (2) the displacement of the piston should be the same as the guitar plates at their most compliant points (near the "center" of the lower bout).

The general solution for the displacement amplitude of a uniform thin circular plate is (Kinsler *et al.*, 2000)

$$y(r) = AJ_0(Kr) + BI_0(Kr), \tag{8}$$

where $y(r)$ is a function of radial distance (r) and J_0 and I_0 are the zeroth order Bessel function and zeroth order modified Bessel function. $K2$ is proportional to frequency and contains material parameters such as density and Young's modulus. Application of boundary conditions determine (1) the ratio of the amplitude factors A and B and (2) the discrete values of K (and $\omega = 2\pi f$) that fit the assumed harmonic solutions. The precise form of K is not needed in this model because the material properties (density, bulk modulus, and the effects due to bracing pattern etc.) are represented by one simple measured parameter, namely, the stiffness constant (k) of the top or back plate measured at the most compliant point near the plate centers.

1. Clamped plate

For a rigidly clamped plate, $y(0) = \dot{y}(0) = 0$, the (0,0) mode is (Kinsler *et al.*, 2000)

$$y(r) = A_1[J_0(3.2r/a) + .055I_0(3.2r/a)], \tag{9}$$

where $y(r)$ is the displacement function of radial distance (r), J_0 and I_0 are the zeroth order Bessel modified Bessel functions, a = plate radius, and the effective plate area is (Kinsler *et al.*, 2000)

$$A_e = 0.310A. \tag{10}$$

The effective mass of a plate is proportional to this effective area, so

$$m_e = 0.310\text{m}. \tag{11}$$

Note that because $J_0 = I_0 = 1$, the maximum amplitude $y(0) = 1.055A_1$.

2. Hinged plate

For a hinged plate, $y(0) = \ddot{y}(0) = 0$ and leads to the (0,0) mode solution

$$y(r) = A_1[J_0(2.01r/a) - 0.099I_0(2.01r/a)]. \tag{12}$$

Because $J_0 = I_0 = 1$, the maximum amplitude is at $y(0) = 0.901A_1$. The displacement $y(r)$ for this hinged plate is shown in Fig. 3. The increase in cavity volume due to this displacement can be pictured by rotating the upper curve 360° around the vertical axis.

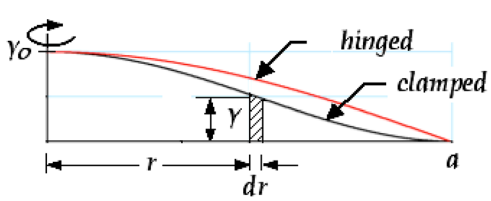

FIG. 3. (Color online) The function $y(r)$ for a hinged plate. The clamped plate is shown for comparison. The shaded area is a cross section of the infinitesimal cylindrical volume element needed to calculate the cavity volume change due to plate displacement. Integration is performed over the radius (r) from the plate center at $r = 0$ to the plate edge at $r = a$.

The cavity volume change is

$$\Delta V_{\text{plate}} = \int_0^a y(r)2\pi r dr. \tag{13}$$

Substituting Eq. (12) into Eq. (13) and integrating gives

$$\Delta V_{\text{plate}} = 0.416A_1\pi a^2. \tag{14}$$

A flat piston equivalent to the plate requires that

$$\Delta V_{\text{plate}} = \Delta V_{\text{piston}}. \tag{15}$$

The piston volume change is just $y(0)A_e$, πa^2 is the plate area, A, and because $J_0 = I_0 = 1$, $y(0) = 1.055A_1$. Combining these with Eqs. (14) and (15)

$$\text{simplifies to} \quad A_e = 0.461A, \tag{16}$$

$$\text{which leads to} \quad m_e = 0.461\text{m}. \tag{17}$$

3. Model effective area and mass

The effective mass and area used in the 4 mass model is the average of the clamped and hinged plate values, i.e.,

$$\bar{A}_e = 0.385A \quad \text{and} \quad \bar{m}_e = 0.385\text{m}. \tag{18}$$

V. MODEL PREDICTIONS

Model predictions for "isolated" oscillators and for the four oscillators interacting under free or clamped boundary conditions are calculated using the two sets of guitar parameters in Table III.

A. "Isolated" oscillators

The model Eqs. (1)–(5) are easily reduced to a single equation in one unknown for each of the four oscillators. A computer program was used to calculate the "isolated"

TABLE III. This table is a complete set of measured and calculated parameters for two guitars. The only parameters taken from resonance experiments are the resistances calculated from measured quality factors using $r = \omega_0/Q$. Units are all SI. Parameter g is defined in Eq. (4)

Guitar	Kohno	D28
Volume (m^3)	0.0124	0.0172
Top mass (kg)	0.034	0.128
Top stiffness (N/m)	40 800	141 000
Top area (m^2)	0.031	0.0375
Resistance (Nm/kg/s)	19.5	32
Back mass (kg)	0.04	0.061
Back stiffness (N/m)	53 000	59 100
Back area (m^2)	0.031	0.0375
Resistance (Nm/kg/s)	22	34
Air piston mass (kg)	0.00051	0.000804
Air piston area (m^2)	0.00589	0.00785
Resistance (N/m)	23	30
Rib mass (kg)	0.233	0.45
Resistance (Nm/kg/s)	30	30
g (N/m^5)	11 400 000	8 240 000

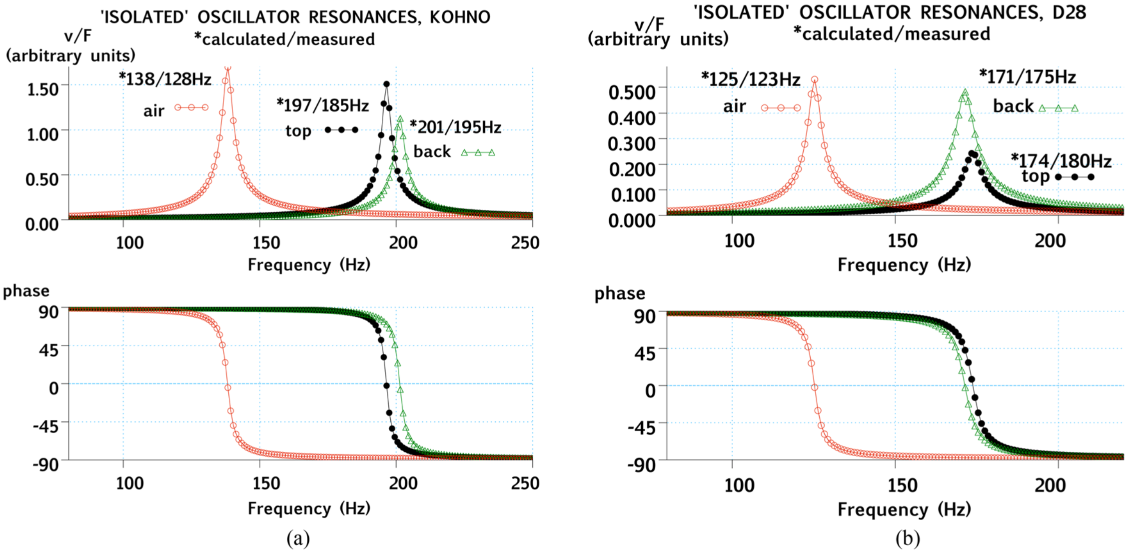

FIG. 4. (Color online) The calculated mobility (v/F) and phase curves for isolated oscillators with calculated and measured resonant frequencies for the classical Kohno (a) and for the steel string D28 (b).

oscillator velocity amplitudes as a function of applied frequency and identify their resonant frequencies and phases. In the top plate this corresponds experimentally to closing the sound hole with a small balsa wood plate and clamping or heavily loading the ribs and the back, driving the top with a harmonic force with constant maximum force amplitude while slowly varying the frequency, and measuring its response with an accelerometer. The top plate was thus isolated from the other oscillators, but affected by both the top plate stiffness and the air trapped inside the guitar cavity. The isolated back and the isolated air piston responses were similarly calculated, plotted, and resonance frequencies compared to experimental values (Fig. 4). These calculated responses are plotted in arbitrary units because a unit force acting on a mass less than a gram for the air piston oscillating in and out of the sound hole is unreasonable. The resonant frequencies were being sought for the isolated resonators, not the relative magnitude of their responses. These calculations are presented to demonstrate the accuracy of the model predicted resonance frequencies and the guitar parameters defined and measured, but are not needed to calculate the coupled oscillator resonances of the whole guitar.

B. Guitar response

The calculated response of each of the four interacting oscillators to a harmonic force of constant maximum amplitude applied to the top mass described in Eqs. (1)–(5) are shown in Fig. 5. Amplitudes are the calculated values relative to each other. Input parameters for these calculations were the same as those used to calculate the isolated oscillator frequencies, but the mass of the ribs was an additional

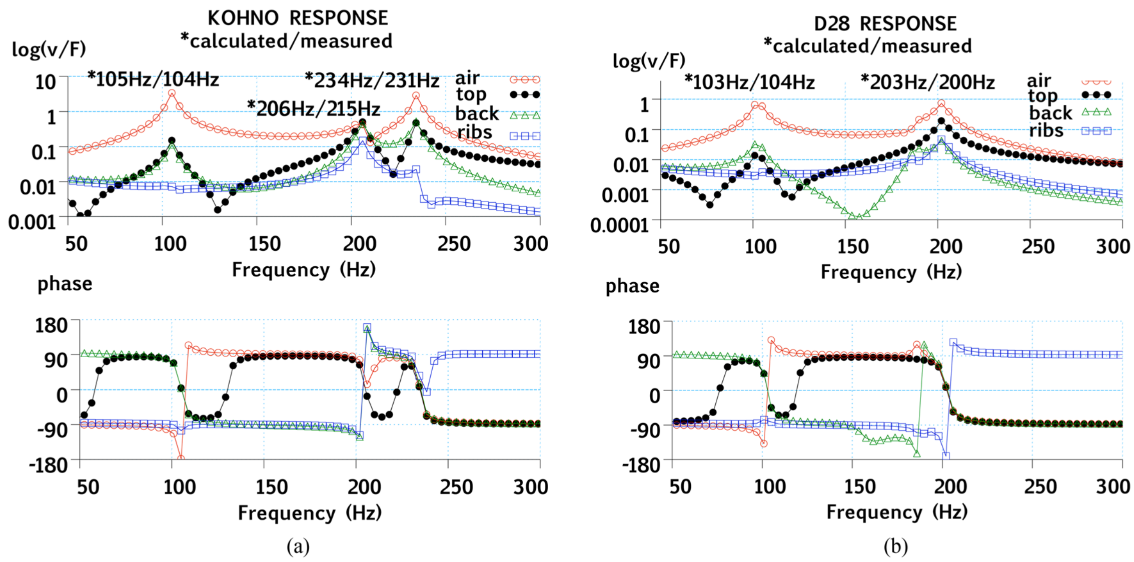

FIG. 5. (Color online) Calculated and measured responses for the Kohno (a) and the Martin D28 (b) are shown. The resonance frequencies are seen to be in substantial agreement with experiments. The relative phases of the oscillators (see also Fig. 1 for definitions of zero phase) at each of the resonances also agree with observation and textbook descriptions (Fletcher and Rossing, 1998). The label "air" stands for the "air piston" described previously.

input. The three resonant frequencies have values and phases consistent with experiment and textbook descriptions (Fletcher and Rossing, 1998). The amplitudes calculated and plotted are their true values relative to each other per unit force applied to the top.

Only two resonances are evident in both the calculated and the measured response of the top plate of this Martin guitar when the four oscillators are allowed to interact freely and the rib mass is between 0.4 and 0.6 kg. This calculated result, unlike the Kohno guitar, is sensitive to the value assigned to the rib mass.

C. Variable rib mass

A Quicktime movie, available from the author and online,[1] shows the Martin D28 response curve as a function of rib mass. The movie and the sequence of response curve frames from it (Fig. 6) show that two resonances (near 100 Hz and near 200 Hz) remain nearly fixed as the rib mass is varied, but the third resonance changes dramatically.

Little or no motion of the sides or neck occurs for two resonances because the top and back plates are in phase (oscillate towards and away from the center of mass together, see D28 response curve and phase) and very little, if any, recoil of the ribs. Thus, the plates couple primarily thru the air. For the lowest resonance, air moves into the cavity when the volume expands (the "breathing" mode) and for the highest of these two, air exits when the volume expands (the "anti-breathing" mode).

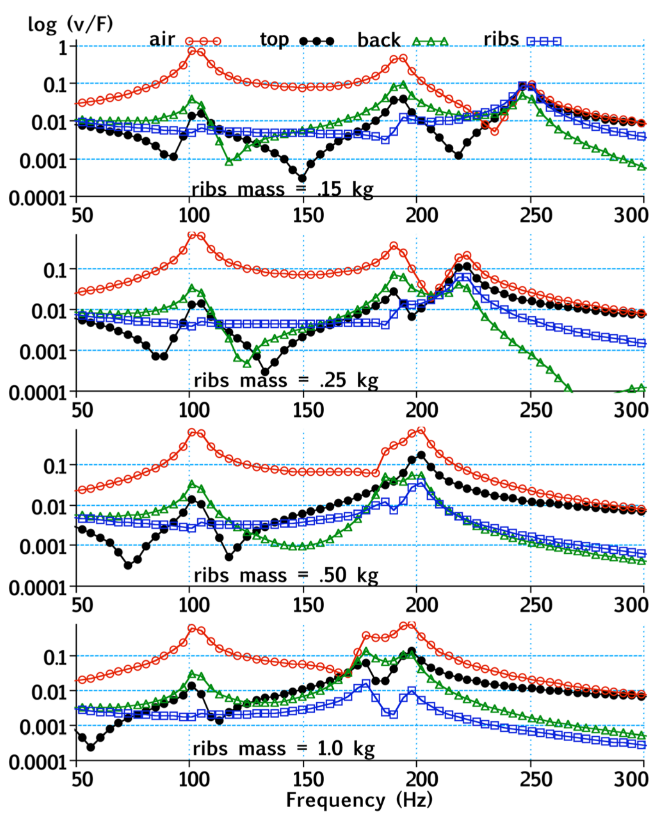

FIG. 6. (Color online) As rib mass is varied, the frequency of one of the (0,0) modes changes from above 250 Hz at very low mass to 180 Hz for large mass. At 0.5 kg, (real, not effective mass) only two resonances occur. The density, area and thickness of this guitar top, back, side, and neck showed that the sides and neck together account for between 1/4 and 1/3 of the total guitar mass, which is near 0.5 kg.

For the third resonance, which changes as rib mass is varied, the top and back couple primarily thru the ribs as they oscillate in the same direction (180° out of phase with each other, see Fig. 1 for definitions of phase). When one moves towards the center of mass, the other moves simultaneously away from it. Momentum is nearly conserved as the ribs recoil, thus causing a strong dependence of this resonant frequency on rib mass (the "bending" mode). As the rib mass varies from 0.15 kg (unrealistically low) to 0.35 kg (a more reasonable value), the frequency of this mode drops from nearly 250 Hz and approaches 200 Hz. Between 0.4 and 0.55 kg this bending or "bar" mode and the higher (0,0) mode are replaced by a new mode with some characteristics of the original two. The amplitude and phase diagrams of Fig. 5(b) show that the top and air piston for the D28 move in the same direction with relatively high amplitude and the back and sides both move together in the direction opposite to the top and air piston. Furthermore, the back and ribs move with the same amplitude as each other (therefor, near zero relative motion) but with far less amplitude than the top or the air piston. The back has a small amplitude because it is acted upon by the low pressure of the cavity air at the same instant that it is pulled in the opposite direction by the ribs.

Three resonances are reduced to just two. This was the case experimentally for this guitar when driven with free sides and neck (ribs) as previous tables summarized. At 0.6 kg and above, this third or bending mode resonance drops below 200 Hz and the anti-breathing mode reappears again at 200 Hz. The response is now that predicted by a three mass model (no rib motion and therefore equivalent to an infinite rib mass) and demonstrated experimentally when the ribs were rigidly clamped. Data was taken from the Quicktime movie of frequency and rib mass for this changing oscillation and a graph and curve fit generated (Fig. 7).

Two unusual features of this guitar may contribute to this behavior. Unlike many other guitars, it had a lighter and

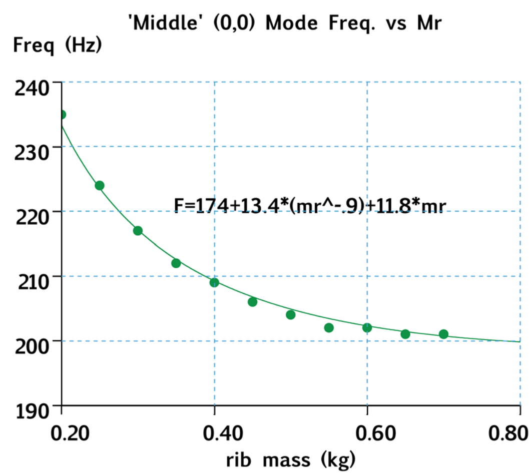

FIG. 7. (Color online) The dependence of the 'bending' mode resonance on rib mass is significant, but less dramatic than the graph suggests at first sight. Real guitars have neck and side masses limited to the center portion of this graph. The D28, for example, has only two resonances for masses between 0.40 and 0.55 kg, and rib masses lower than this or much higher do not occur in real guitars. The model predicts unreasonably large rib motion at rib masses below .2 kg and fails to be a realistic representation.

softer back than top, and it had a bar mode (whole guitar resonance with strong neck vibrations) near 200 Hz. The second guitar modeled in this way, the Kohno classical, showed three (0,0) resonances for all reasonable values of rib mass. It had, more typically, a more massive and stiffer back than top, and its first two bar-like modes were both measured (and calculated) to be far from 200 Hz.

VI. GUITAR BAR MODES

If the model predicts that rib masses between 0.40 kg and 0.55 kg eliminates one resonance near 200 Hz, it is reasonable to assume that a reactive load like a neck and body resonating like a bar at this same frequency might be an alternative way to look at this mode. Bar-like modes, therefore, for the whole guitar were measured for three guitars and are shown in Fig. 8. A simple model was used to calculate bar modes for the Kohno classical guitar and compared to the measured bending mode. This guitar had a spruce top, rosewood sides and back, and neck made of a sandwich of ebony (fret board) and mahogany.

A. Measured bar modes

Measurements of bar modes were made with sound posts wedged between the top and back plates, cavity filled with small styrofoam spheres, sound hole closed, rubber band supports, and driven at the tail block. Table IV lists the measured bar modes for three guitars and calculations for the Kohno classical guitar.

B. Calculated bar modes

The frequencies of the first few normal modes for a free-free bar are (Kinsler *et al.*, 2000)

$$f = \frac{\pi k}{8L^2}\sqrt{\frac{Y}{\rho}}(3.0112^2, 5^2, 7^2, \ldots.), \tag{19}$$

where k =radius of gyration (second moment) of an area, Y = Young's modulus, and ρ = density.

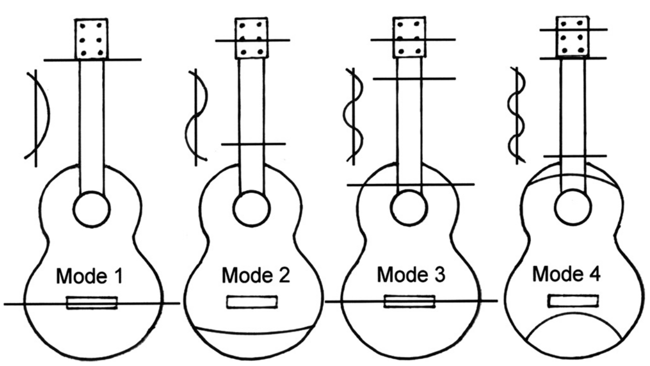

FIG. 8. Location of nodes in the first few bar modes of guitars tested. The sign like curves through the vertical lines to the left of the guitar images represent the shape of a simple bar in a similar mode of oscillation. Nodes occur where the curves intersect the verticals and correspond to the nodal lines in the guitar images. They represent the shape of the entire length of the guitar, not just the neck.

TABLE IV. Measured bar modes sketched in FIG. 9 for three guitars. The calculated frequencies match measured values for classical guitars for the first two modes surprisingly well. At higher frequencies, the guitar sound box most certainly distorts in a way far different than a simple bar. The D28 differs from the others because it had a steel bar in the neck.

	Guitar "bar" mode frequencies (Hz)			
Guitar	Mode1	Mode 2	Mode 3	Mode 4
MartinD28	55	198	408	542
Kohno	91	272	353	…
Krempel	90	302	350	…
Calculated	84	235	579	…

If a guitar neck is modeled as a composite of a rectangular fret board and a "rectangular" neck, Eq. (19) may be replaced by the approximation

$$f = \frac{\pi}{8L^2}\left[\frac{t_f}{\sqrt{12}}\sqrt{\frac{Y_f}{\rho_f}} + \frac{t_n}{\sqrt{12}}\sqrt{\frac{Y_n}{\rho_n}}\right](3.0112^2, 5^2, 7^2, \ldots.). \tag{20}$$

where f = fret board, n = neck, and L = total length of the guitar neck and body. TR and t_n are the thickness of the rectangular cross section. The guitar body examined have an average mass per unit length roughly the same as the neck.

The second bar mode resonance measured at 198 Hz for the D28 guitar matches the second (0,0) mode frequency predicted by the four-mass model. The mode shapes of this one resonance are the same for these two models.

VII. CONCLUSION

The four-mass model is a straight forward extension of and modest improvement over past models. It is more successful, especially in accounting for the possible loss of one resonance, in the low frequency range where (0,0) modes occur. Considering its simplicity, the model predicts both isolated oscillator and coupled oscillator resonances with surprising accuracy.

The static stiffness, measured over a relatively small area (1 square cm), is perhaps the most extreme example of adopting a quite simple method of determining a lumped parameter value for simplifying rather complex underlying physics. If it is justified to replace a distributed plate mass with a lumped mass only 38.5% of its true value [Eq. (18)], then it is reasonable to expect that the effective stiffness must be reduced too for similar reasons. This justification for using the static stiffness measure in the model is admittedly weak. A better defense of its use is that it works, i.e., the calculated resonances match the measured values.

When a guitar exhibits three (0,0) modes, as the Kohno classical guitar does, one of them may be modeled in two quite different ways; (1) the four-mass model that shows the top and back plates, coupled by the ribs, oscillating in the same direction and with the sides and neck recoiling and creating a bending mode of the entire guitar, or (2) as a free-free bar in its second mode of oscillation. If a guitar exhibits just two (0,0) modes, the higher mode may, like the D28 guitar, be described as similar to the anti-breathing mode but

with little motion of the back relative to the sides. The two mode case may be modified to create three (0,0) modes by adding mass to the neck, the sides, and/or the tail block. Adding mass to the neck will clearly lower the second whole guitar bar mode frequency and reduce the amplitude of neck vibrations.

The four-mass model offers the opportunity to explore the effects of guitar parameter changes on the placement of these resonances, and explains how to avoid the loss of one of them if desired. The direct determination of fundamental parameters like plate stiffness and effective mass and area has been demonstrated. These and other parameters are easily modified and their effects studied using software for Apple computers and a PC version (both freeware) available from the author. They can be used to predict the behavior of specific guitars and of parameter changes on guitars generally and hopefully will contribute, as other guitar acoustics research has, to improved guitar design (Eban, 1998).

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[1]See supplementary material at E-JASMAN-130-032291 for a QuickTime movie showing the dependence of one (0,0) mode resonance on rib (neck and sides) mass titled D28Movie.qt, and a related readme.txt file.

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